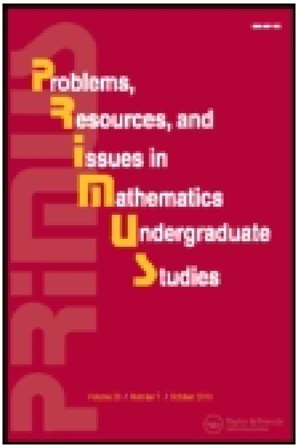

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Note Taking in Multi-Media Settings

Kelly Black and Guangming Yao

Abstract: We provide a preliminary exploration into the use of note taking when combined with video examples. Student volunteers were divided into three groups and asked to perform two problems. The first problem was explored in a classroom setting and the other problem was a novel problem. The students were asked to complete the two questions. Furthermore, the students were randomly assigned to three groups and given a set of handouts. One group did not watch the video. One group watched the video but did not take notes. The final group watched the video and took notes. We then used a coarse scoring system to provide a rough estimate of the students' performance on the two questions. We made use of the students' test scores prior to the exercise as a way to provide a covariate to compensate for student aptitude in this subject. We found that the group of students solving the first, familiar problem demonstrated no statistical difference in their performance. There was a statistical difference between the groups who watched the video and did not watch the video for the novel problem, but there was not a statistical difference between students who did not take notes while watching the video.

Keywords: Video instruction, ordinary differential equations, notes, flipped classroom.

1. INTRODUCTION

Many students place a high value on the activity of writing their notes [6]. We seek to determine if the activity of note taking translates into a beneficial practice when short, Internet-based lecture videos are available.

Note taking itself has been the focus of numerous studies. The investigations indicate students take part in a wide range of tasks while taking notes, and writing notes in class and reviewing those notes, either in class or afterward, has a positive impact on students' performance [16]. Students recall more lecture materials if they record it in their notes [5]. Students who take

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notes score higher on both immediate and delayed tests of recall and synthesis than students who do not take notes [12]. Moreover, the more students record, the more they remember and the better they perform on exams [10].

The process of taking notes is not the only factor in student's recall, though. For example, Kiewra et al. [12] found that students who take notes but do not review them earn lower exam scores than students who review notes prior to the exam. Additionally, students not present at the lecture but given notes to review (either the instructors' notes or notes taken by other students) did almost as well as the students who reviewed their own notes and significantly better than students who did not review.

Other studies have revealed that the accuracy and quality of the notes can vary, and the process of taking notes does not necessarily result in higher performance on achievement measures [6], and the resulting quality of the student's notes can impact the students' performance [14]. Furthermore, there is a relationship among the notes and the test problems. Note takers perform slightly worse than students who do not take notes if the problems on the exams are not closely related to the materials in the notes with respect to the students' point of view [15].

One thing is clear: students who take an active role in classroom activities are better able to synthesize ideas and use them in deeper ways to solve related problems [13, 21]. One way to achieve the goal of making classroom activities more meaningful and keeping students engaged is through the use of a "flipped classroom" [1, 17]. We have begun to explore the possibility of conducting a "flipped" classroom for the Introduction to Ordinary Differential Equations class at Clarkson University.

The format includes the use of videos that provide examples and explanations. It is not a new idea and is a topic that was explored in a similar study by Peper and Mayer [15]. The role of examples and how to employ them have also been explored by others [19]. Our goal is to simply ascertain student performance on relatively simple tasks on familiar versus novel topics, and one question we seek to explore is what advice/instructions to provide to the students as they make use of various resources. In particular should we recommend that the students take notes or focus on the video as it plays? There is no one answer for all students [9], but insight into large-scale behaviors can provide insight.

It has been our experience that some students have an intense need to take notes in our classroom. Even when notes are made available to them many students insist on taking their own notes to supplement the notes provided by the instructors. The process of taking notes can create time constraints on students [16]. Instructors have to pause during class to make sure that students can catch up in their writing as well as ensure that students can pay attention to the key ideas.

To explore the role of note taking in the context of video instruction we asked students taking part in a large lecture introductory ordinary differential equations class to attempt two problems. The specific problems are given in Appendix A. The first problem is a relatively straight-forward, first-order, initial value problem to be solved using the Laplace transform. The second problem required that the students use the convolution and the Laplace transform.

The students were randomly assigned to one of three groups. One group did not watch the videos, one group watched the video without taking notes, and the third group took notes while watching the videos.

We compared the groups using Analysis of Covariance (ANCOVA), with the students' scores on prior course tests as the covariate. We found that there was not a significant difference among the three groups of students with respect to the familiar problem. There was a statistical difference between the students who did not watch the video and the other groups of students for the novel problem, but the two groups of students who watched the video did not demonstrate a statistical difference.

2. THE EXPERIMENT

The specific details of the experiment are given here with separate subsections providing details about the students, the way the students were divided into their three groups, and the potential problems with this experiment.

2.1. The Students

Every student in the second year Introduction to Ordinary Differential Equations course was asked to take part in the experiment. The majority of the students have majors declared in engineering disciplines (Engineering and Management, Chemical, Aeronautical, Civil, Computer, Electrical, Environmental, Mechanical, and Software Engineering) with a minority of students with majors in Physics, Mathematics, Computer Science, or University Studies. Table 1 includes a breakdown of all of the students' first major in the entire class as well as the number of students for each major who took part in the experiment.

The students took part in three large lectures a week, and they had one additional recitation with an undergraduate teaching assistant each week. Each recitation had a mix of students from all three lecture sections, and they enrolled in the class that best met their schedule. Because of the common recitations we are required to make every effort to coordinate the lectures to a high degree of consistency.

The instructors used the same class materials and assignments, and the materials are freely available [2]. The resulting experiment allows for a large number of participants and yields a higher power on our statistical tests that is

Table 1. Distribution of students by their declared first major. The second column contains the total number in the class. The third column contains the number of students taking part in this exercise

Major	Total number	Participants
Aeronautical Engineering	49	26
Biomolecular Science	1	1
Chemical Engineering	14	8
Chemistry	4	1
Civil Engineering	17	10
Computer Engineering	18	11
Computer Science	5	4
Digital Arts and Sciences	1	0
Electrical Engineering	27	16
Engineering & Mgmt	34	20
Engineering Studies	5	3
Environmental Engineering	6	5
Environmental Sci & Policy	1	1
Financial Info & Analysis	1	1
Mathematics	6	2
Mechanical Engineering	115	66
Physics	4	2
Software Engineering	9	5
University Studies	1	1
Total:	318	183

rarely found in the literature. The wide variation in the way students take notes [6, 11] and the quality of the notes produced [6, 14] should have a lower impact on our statistical results.

2.2. The Tasks

We asked the students to complete two tasks which are given in Appendix A. Two examples were shown in separate videos. The students were asked to watch one video and then finish the corresponding tasks, which can improve the effectiveness of the students' notes [7]. The videos were hosted on YouTube [8], which allowed the students to experience the videos in their personal environments, and the students have complete control on pausing and replaying the videos.

The first task was to determine the solution to an ordinary differential equation using the Laplace Transform. Similar problems were discussed in the classroom, and the problem itself was relatively straight-forward. This first problem is referred to as the "familiar" problem. The goal in assigning this first problem was to assess the students' abilities to complete relatively straight-forward tasks [19], to establish a base-line for the class, and to provide a basic test of the student skills and their approach to this exercise [14].

The second problem was a new topic that had not been discussed in the classroom. The problem provided two barriers to the students. The first barrier was the idea of a convolution and its relationship to the Laplace Transform. The second barrier was the requirement to provide a careful description of the solution *procedure*.

All students were given a set of notes describing the basic idea (see Appendix B). One set of notes was provided for the first problem, and a separate set of notes was provided for the second problem. All of the students were asked to look over the notes first, and then watch the videos. Our goal was to mimic the approach used in a flipped classroom and provide a similar set of supplemental materials while limiting issues associated with other types of hypermedia [14].

The students were randomly assigned to one of three groups using a script [20], see Figure 1. The students were assigned to one of the three groups with an equal probability (one-third) for each group. There was no guarantee that students would complete their assigned tasks as requested, so statistical tests were chosen in advance that did not require equal numbers of students in each group. To minimize misunderstandings we stated the expectations for each group in class, in an email, and in the materials given to the students. Our expectations and the general procedure for students to follow are described in the statement.

The three groups are shown in the diagram in Figure 1. Students in the first group, (A), were asked to try to solve the problems without the benefit of watching a video. Students in the second group, (B), were asked to watch the videos [3, 4] without taking notes and then solve the problems. Students in the third group, (C), were asked to watch the videos [3, 4] and take notes while watching the video and then solve the problems. The students were given no training nor was any discussion provided with respect to approaches to employ in taking notes [6, 11, 12].

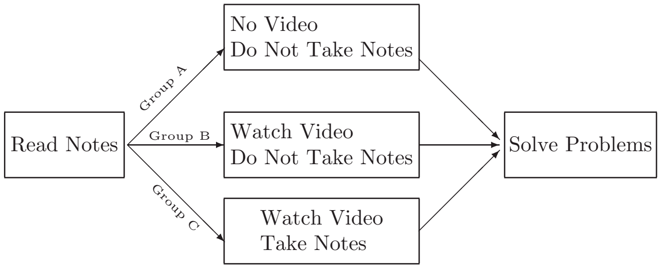

Figure 1. Every student was asked to read the notes. The difference is whether or not a student watched the video and if she or he took notes.

Table 2. The scoring rubric for determining the score for an individual problem

Score	Rationale
5	A perfect paper with maybe one small algebra mistake
4	The student got the idea but had several small mistakes that kept the student from getting the correct solution
3	The student started the problem but had some difficulties in either algebra or other idea that made the student miss the problem
2	The student was able to start the problem but had considerable difficulties
1	The student simply did not get the problem at all

The students' final step was to work through two problems. The problems were different from the examples but were similar in nature to the examples in the videos. The solutions were then collected in class on the same date.

The cover sheet on the materials given to the students included an overview of our goals and a rationale. The cover sheet had check-boxes for the students to identify their assigned groups. If the students checked a group that was different from his or her originally assigned group then the paper was discarded. Additionally, the students were asked to describe how they used the videos during the process. If the description was not consistent with the group assignment then the paper was discarded. Less than 10 papers were discarded for these reasons.

The remaining papers were then evaluated by the two authors. The evaluation was done separately, and the results were compared. The papers that were given different scores were then compared, and the papers were reviewed by the two authors together. If we could not agree on a score for a problem then the student's work was discarded, and two papers were removed from consideration.

Each problem was given a score on a scale of one to five. The details of the scoring rubric are given in Table 2. A score of five indicated that the student completed the paper with no mistakes. A score of one indicated that the student did not understand the question. The statistical measures employed are used to detect differences in the scores so using a one rather than a zero for the base is immaterial.

2.3. Potential Problems and Limitations

A primary issue is that this study was part of a voluntary exercise for extra credit in a class. This was done as a way to ensure a large number of participants but makes it difficult to draw firm conclusions. There is no way to decide how much effort the students gave in this exercise. This may result in a bias toward more motivated students who may be more likely to have taken part in the exercise.

Another potential issue is that some students counted in the A group may have watched the videos. The student notes indicate that some students did feel pressure to do well and give a complete answer on the assignment. A small number of students, less than 10, also indicated in their notes that they felt frustrated at not being able to complete the problem and said that they eventually watched the videos. The papers associated with these statements were removed from consideration, but there is no way to tell how many other students acted in this manner.

It is important to note that the statistical methods and the experiment are designed to determine if there are differences among the three groups. The potential problems serve to diminish the differences, and they serve to make it more difficult to determine if there are differences among the groups. In this sense the situations that do show a difference between the groups do so in the face of an experiment that makes it more difficult to detect those differences.

3. THE ANALYSIS

Here we examine the statistical results from the experiment. The results are explored in two different ways. First, the frequencies for the scores were compared across the three groups. Two-way tables between the groups and the student scores were constructed and compared. The number of students and the nature of the results are not appropriate to use a χ^2 test because of the low numbers of students scoring a are on the problems. Second, the scores for each problem are compared using ANCOVA.

Prior studies have indicated that students' abilities can impact the quality of their notes [14, 15], and we make use of a covariate to control this factor. Prior to the assignment the students had taken three tests. The material on the third test was closely aligned to the material of this exercise. The students completed the assignment before the end of the course but after the students took the third test. Since a test occurred after the assignment being examined the test would have violated the independence assumptions inherent in ANCOVA. Thus, the third test and final scores in the course were not used as a covariate in the analysis. The arithmetic mean of the previous two tests were used as a covariate.

All calculations and graphical representations were completed using R [18]. The significance levels used for all decisions is 5%.

4. THE RESULTS

We provide the two-way frequency tables for the students' scores first, and then provide the results of ANCOVA. The results are given separately for the

Table 3. Number of students who took part in the exercise

	(A)	(B)	(C)
Class totals	98	102	118
Participating	56	64	64

problems. We also provide results for ANCOVA on the second problem for just the (B) and (C) groups.

First, the totals for the groups are given in Table 3. The table gives the total number of students assigned to each problem in the first row, and the number of students who completed the assignment in the second row. The participation rate for the (C) problem was less than the others. However, a χ^2 test for the frequencies yields a value of $\chi^2 = 0.438$ with two degrees of freedom, which results in a p-value of approximately 80%, so we cannot state that there was a large difference in the participation rates.

We now turn our attention to the results of the exercise. The two-way frequency table for the first problem is given in Table 4. The numbers of students in the lower categories even when combined on the low end are very small compared with the size of the full group. Hence, the results are not consistent with the assumptions for a χ^2 test, so we only provide the raw results along with the results from ANCOVA.

The results for the first problem for the different groups are shown in Figure 2. Most of the students received high scores for the first problem across all groups. There are a small number of students in the (A) group who did not do well, but this group of students is relatively small compared with the group as a whole.

The two-way frequency table for the second problem is given in Table 5. Again, the frequencies are small, and the results are not consistent with the assumptions for a χ-squared test. In this case the raw percentages provide larger differences.

Table 4. Two-way frequency table for the students' results on the first problem

	(A)	(B)	(C)	Total (Percentages (%))
1	4	0	0	4 (2)
2	5	5	1	11 (6)
3	5	7	10	22 (12)
4	9	9	8	26 (14)
5	33	43	45	121 (66)
Total	56	64	64	184
Percentages (%)	30	35	35	

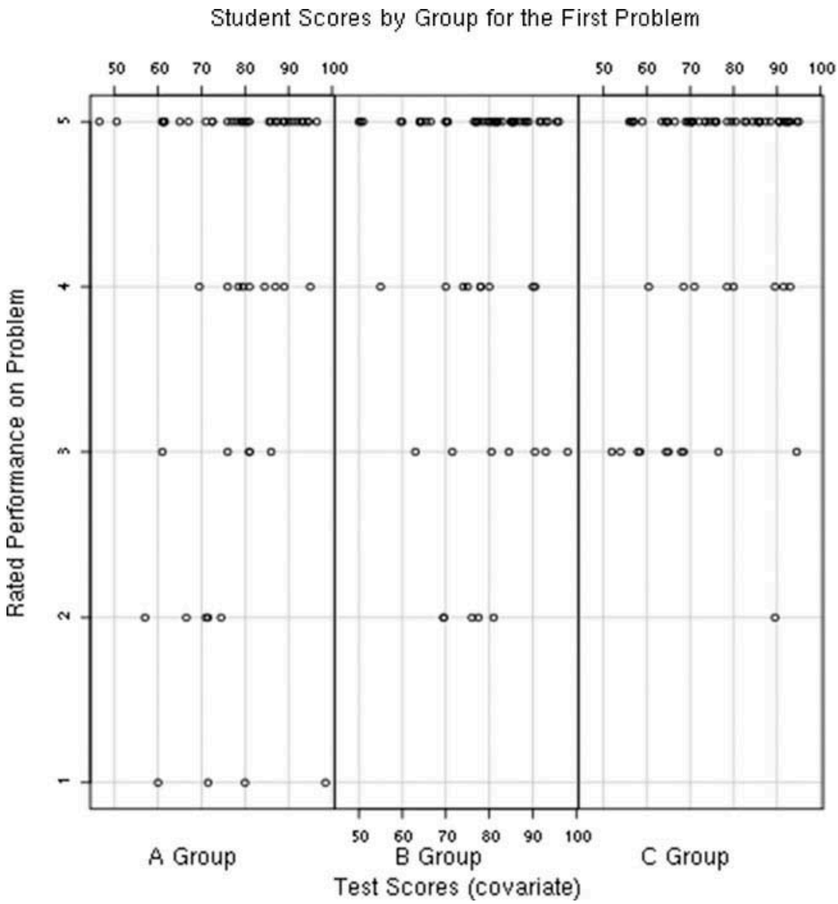

Figure 2. Student performance ratings for the first problem by groups as a function of their test scores. Each column is for the (A), (B), and (C) groups.

Table 5. Two-way frequency table for the students' results on the second problem

	(A)	(B)	(C)	Total
1	18	5	7	30
2	2	4	1	7
3	2	3	5	10
4	5	4	1	10
5	29	48	50	127
Total	56	64	64	184

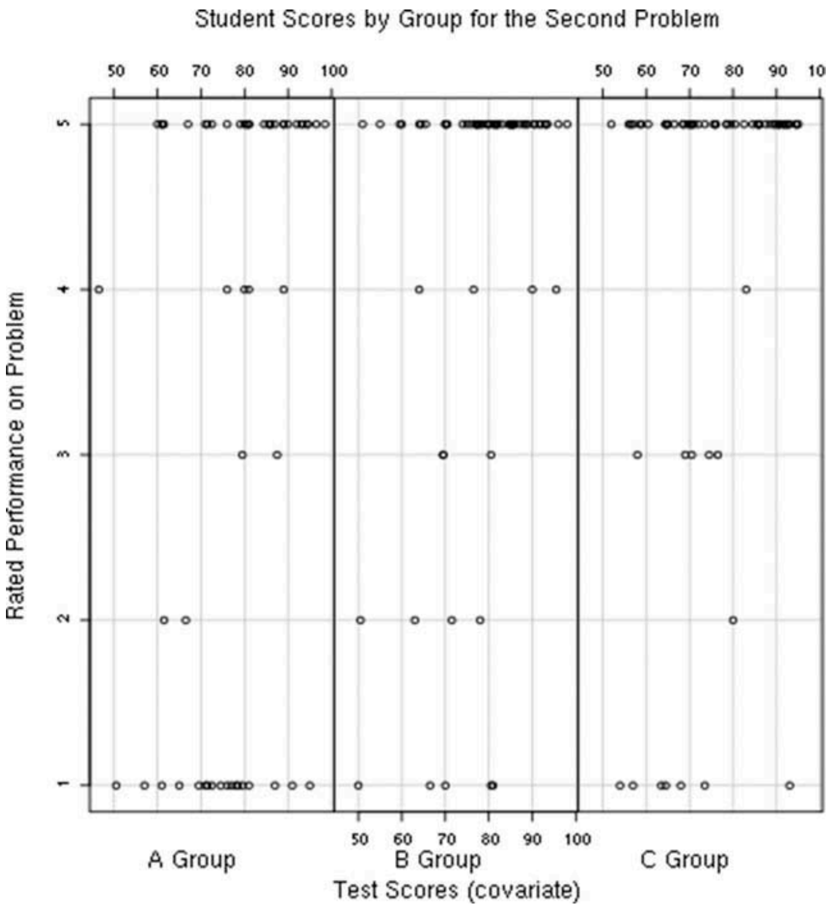

Figure 3. Student performance ratings for the second problem by groups as a function of their test scores. Each column is for the (A), (B), and (C) groups.

The results for the second problem for the students divided by their group assignment is shown in Figure 3. Again a large number of students did well on the problem. In this case, though, there is a larger proportion of students in the (A) group who did not perform well. In particular there is a large number of students at the lowest rating.

An ANCOVA test was employed to determine if there are differences in the performances on the two problems between the students in the three groups. For the first problem, a topic covered in class, the results are shown in Table 6. Interestingly, there was not a strong difference in performance on that problem based on the test scores with an F-score of 1.92 and a p-value of 16.7%. The resulting differences across groups with the test scores as a covariate also did not show a significant difference with an F-score of 2.64 and a p-value of 7.4%.

Table 6. ANCOVA results for the first problem with the students previous test performance as a covariate

	Df	Sum Sq	Mean Sq	F value	Pr $(>F)$
Test avg. deviation	1	2.054	2.0542	1.9224	0.1673
Group (ABC)	2	5.646	2.8228	2.6417	0.0740
Residuals	180	192.338	1.0686		

Table 7. ANCOVA results for the second problem with the students previous test performance as a covariate

	Df	Sum Sq	Mean Sq	F value	Pr $(>F)$
Test avg. deviation	1	32.12	32.122	15.710	0.0001063
Group (ABC)	2	33.92	16.960	8.295	0.0003581
Residuals	180	368.04	2.045		

Table 8. ANCOVA results for the first problem with the students previous test performance as a covariate. These results are limited to scores from groups (B) and (C)

	Df	Sum Sq	Mean Sq	F value	Pr $(>F)$
Test avg. deviation	1	0.790	0.78968	0.9819	0.3236
Group (BC)	1	0.487	0.48702	0.6056	0.4379
Residuals	125	100.52	0.80422		

The results for the second question were quite different. The results from the ANCOVA are given in Table 7. The test scores appear to have a role in how the students performed on the second problem, with an F-score of 15.7 and a p-value of less than 0.1%. The results using the test scores as a covariate resulted in a significant difference among the groups with an F-score of 8.30 and a p-value of less than 0.1%.

The next question to explore is whether or not there is a difference in student performances between just the (B) and the (C) groups. Both groups watched the video. The only difference is that the (C) group took notes. The results for the two groups are given in Table 8 and Table 9.

The results from just group (B) and (C) for the first problem are given in Table 8. The test average did not play a significant role with an F-score of 0.98 and a p-value of 32.3%. There was not a significant difference among the performances on the first problem by students in the two groups when using the test score as a covariate with an F-score of 0.61 and a p-value of 43.8%.

Table 9. ANCOVA results for the second problem with the students previous test performance as a covariate. These results are limited to scores from groups (B) and (C)

	Df	Sum Sq	Mean Sq	F value	Pr $(> F)$
Test avg. deviation	1	20.066	20.0664	12.6262	0.0005375
Group (BC)	1	0.150	0.1500	0.0944	0.7591637
Residuals	125	198.659	1.5893		

The results from the (B) and (C) groups for the second problem are given in Table 9. In this comparison the test average did play a significant role with an F-score of 12.63 and a p-value of less than 0.1%. There was not a significant difference between the performances on the first problem by students in these two groups. When using the test score as a covariate, however, the result is an F-score of 0.09 with a p-value of 75.9%.

5. CONCLUSIONS

The growing role of different media impacts our students and impacts how we interact with them. We seek to gain insight into the use of different media combined with the role of note taking. We wish to develop appropriate instructions for our students for the future employment of a flipped classroom. This study presents us with insight into how our students interact and understand material that is given in mixed-media presentations.

We divided our students into three groups and presented them with two problems. For the first problem, a familiar question, there was not a significant difference in performance ratings across the three groups. Moreover, the students' scores on the first problem were not strongly influenced by the covariate, their previous test scores. On the second problem there was a statistically significant difference in performance ratings across the three groups with a p-value of less than 0.1%. In this case the covariate did play a strong role in the students' performance ratings.

When the analysis was limited to just the groups that watched the video there was not a statistically significant difference between performance ratings. Also, for this case, the covariate did not strongly influence the results for the first problem but did influence the results on the second problem.

As noted in Section 2.3 there are some deficiencies in this experiment. These results offer evidence that the way the students interact with diverse media matters with respect to our performance metric, and it offers insight into how to develop the instructions provided to the students when using video instruction. Moreover, this has purely been a quantitative view into the question and does not offer insight into student learning or how the students perceive the use of videos.

Further examination with a qualitative assessment into the students' understanding of the material is necessary before reaching conclusions. Our study cannot conclude a simple answer to the question of whether or not is it efficient to take notes while video materials are available, but it challenges our students' common wisdom that the process of note taking itself is an important part of learning the material. The easy availability of alternate media, such as videos, makes it important to question basic assumptions about how we ask our students to interact with the materials.

APPENDIX A: THE QUESTIONS

The text for the two questions are given below:

Problem 1. Use the Laplace Transform to determine the solution to the differential equation

$$y' + 5y = t,$$

$$y(0) = 1.$$

Problem 2. Use the Laplace Transform to show that

$$\int_0^t e^{-\omega} \cos(\omega) d\omega = -\frac{1}{2} e^{-t} \cos(t) + \frac{1}{2} e^{-t} \sin(t) + \frac{1}{2}.$$

APPENDIX B: NOTES PROVIDED TO THE STUDENTS

Overview

These notes are provided to supplement class notes and provide an extra resource for two related topics. The topics revolve around the use of the Laplace Transform to solve linear differential equations as well as a way to use the Laplace Transform to solve a particular class of integrals using the convolution theorem.

The first topic is a broad overview of how to use the Laplace Transform to determine the solution to a linear differential equation. The Laplace Transform is defined and an identity is derived for the Laplace Transform of the derivative of a function.

The second topic is an introduction to the convolution. The basic idea is presented, and the convolution theorem is given. A broad overview is given on how to determine the solution to one particular class of integrals.

Solving Linear Differential Equations with the Laplace Transform

The Laplace Transform is used to transform a differential equation into a form that requires the use of algebra to determine the solution with minimal knowledge of calculus with respect to the solution technique employed. The Laplace Transform is defined in terms of an improper integral as follows:

Definition 1. *The Laplace Transform of a function, $f(t)$, is defined to be*

$$\mathcal{L}\{f\} = \int_0^\infty f(t)e^{-st}dt.$$

The Laplace Transform of a differential equation, if it exists, can be used to determine the solution to a linear differential equation. Before the solution technique for differential equations is provided an example of how to find the Laplace Transform of a given function is explored:

Example 1. *Determine the Laplace Transform of $f(t) = t$.*

First, from the definition of the Laplace Transform the improper integral,

$$\mathcal{L}\{t\} = \int_0^\infty te^{-st}dt,$$

must be determined. Because it is an improper integral it can be written as a limit,

$$\mathcal{L}\{t\} = \lim_{M\to\infty}\int_0^M te^{-st}dt.$$

This integral can be transformed by integrating by parts. Here we let $u = t$ and $dv = e^{-st}$. From this definition we get

$$\begin{aligned}\mathcal{L}\{t\} &= \lim_{M\to\infty} t\cdot\frac{-1}{s}e^{-st}\Big|_0^M - \int_0^M 1\frac{-1}{s}e^{-st}dt,\\ &= \lim_{M\to\infty} t\cdot\frac{-1}{s}e^{-st}\Big|_0^M - \frac{1}{s^2}e^{-st}\Big|_0^M,\\ &= \lim_{M\to\infty}\frac{-M}{e^{sM}} - \frac{1}{s^2}e^{-sM} + \frac{1}{s^2}.\end{aligned}$$

If the value of s is greater than 0, then the first term in the expression can be transformed using L'Hôpital's rule. In this case the first two terms go to zero as M gets large. The result is that $\mathcal{L}\{t\} = \frac{1}{s^2}$.

Table of Laplace Transforms

A library of Laplace Transforms can be constructed. In the table below the Laplace Transform of a wide variety of functions have been evaluated. The Laplace Transform of the functions is listed, and the the table can be read from left to right to get the Laplace Transform of a given function. The table can be read from right to left to go backwards and determine the function whose Laplace Transform is given to you.

Properties of the Laplace Transform:

1. $\mathcal{L}\{c_1 f(t) + c_2 g(t)\} = c_1\mathcal{L}\{f(t)\} + c_2\mathcal{L}\{g(t)\}$
2. $\mathcal{L}\{f\} = F(s), \mathcal{L}\{tf(t)\} = -F'(s), \mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$
3. $\mathcal{L}\{f\} = F(s), \mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(\sigma)\, d\sigma$

Table of Laplace Transforms:

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\left\{e^{at}\right\} = \frac{1}{s-a}$$

$$\mathcal{L}\{\cosh(at)\} = \frac{s}{s^2-a^2}$$

$$\mathcal{L}\{\sinh(at)\} = \frac{a}{s^2-a^2}$$

$$\mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2+\omega^2}$$

$$\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2+\omega^2}$$

$$\mathcal{L}\left\{e^{at}f(t)\right\} = \mathcal{L}\{f\}(s-a)$$

$$\mathcal{L}\{f * g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$$

$$f * g = \int_0^t f(t-u)g(u)du$$

$$\mathcal{L}\{u_a(t)f(t-a)\} = e^{-as}\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\left\{y'(t)\right\} = -y(0) + s\mathcal{L}\{y(t)\}$$

$$\mathcal{L}\left\{y''(t)\right\} = s^2Y(s) - sy(0) - y'(0)$$

$$\mathcal{L}\{\text{step}(t-a)f(t-a)\} = e^{-sa}F(s)$$

Using the Laplace Transform to Solve a Linear Differential Equation

There is one property of the Laplace Transform that makes it ideally suited to determine the solution to a linear differential equation. An identity is found by expressing the Laplace Transform of the derivative of a function in terms the Laplace Transform of the function itself.

Theorem 1. *The Laplace Transform of y' is*

$$\mathcal{L}\left\{y'\right\} = -y(0) + s\mathcal{L}\{y\}.$$

To show this identity first use the definition of the Laplace Transform, from Definition 1, for the Laplace Transform of the derivative,

$$\mathcal{L}\left\{y'\right\} = \int_0^\infty y'e^{-st}dt.$$

This expression can be transformed using integration by parts. If you let $u = e^{-st}$ and $dv = y'$ the expression becomes

$$\begin{aligned}
\mathcal{L}\left\{y'\right\} &= \int_0^\infty y'(t)e^{-st}dt, \\
&= \lim_{M\to\infty} \int_0^M y'(t)e^{-st}dt, \\
&= \lim_{M\to\infty} y(t)e^{-st}\Big|_0^M + \int_0^M se^{-st}y(t)dt, \\
&= \lim_{M\to\infty} y(M)e^{-sM} - y(0) + s\int_0^M y(t)e^{-st}dt.
\end{aligned}$$

If we assume that the function is $y(t)$ the solution to a linear equation then it cannot grow faster than a linear exponential if the forcing function does not grow faster than a linear exponential. If this is the case a value of s can be found that is large enough so that the first term of the expression goes to zero. The last term in the expression is the Laplace Transform of the function, y,

$$\mathcal{L}\left\{y'\right\} = -y(0) + s\mathcal{L}\left\{y\right\}. \tag{1}$$

To determine the solution of a linear differential equation find the Laplace Transform of the entire equation. Use the identity in equation 1 to express the equation in terms of $\mathcal{L}\{y\}$. Follow the algebraic steps necessary to isolate the Laplace Transform of y, and determine y by finding the inverse Laplace Transform of the resulting expression.

The Convolution

The convolution of two functions, $f(t)$ and $g(t)$, is a way to define a general method of "averaging" a function based on the values in a previous time period. It is given in the following definition:

Definition 2. *Given two functions, f and g, their convolution is defined to be*

$$f * g = \int_0^t f(t-w)g(w)\,dw.$$

Example 2. *Find the convolution of* $f(t) = 3t$ *and* $g(t) = 5t - 1$.

From the definition the convolution is given by

$$\begin{aligned} f * g &= \int_0^t 3(t-w)\cdot(5w-1)\,dw, \\ &= \int_0^t 3(5wt - t - 5w^2 + w)\,dw, \\ &= \int_0^t 15wt - 3t - 15w^2 + 3w\,dw. \end{aligned}$$

To determine the value of the integral treat t as if it is a constant:

$$\begin{aligned} f * g &= \frac{15t}{2}w^2 - 3tw - 5w^3 + \frac{3w}{2}\bigg|_0^t, \\ &= \frac{15t^3}{2} - \frac{3}{2}3t^2 - 5t^3 + \frac{3t}{2}. \end{aligned}$$

The Laplace Convolution Theorem

The convolution is an operation on two functions, and it is a useful operation in a wide variety of circumstances. In the context of differential equations it has a very nice property with respect to the Laplace Transform. The Laplace Transform of the convolution of two functions is given in the following theorem:

Theorem 2. *The Laplace Transform of the convolution of two functions is*

$$\begin{aligned}\mathcal{L}\{f * g\} &= \int_0^\infty \int_0^t f(t-w)g(w)\,dw\,dt,\\ &= \mathcal{L}\{f\} \cdot \mathcal{L}\{g\}.\end{aligned}$$

The proof of the theorem requires some knowledge of multiple integrals which is a topic covered in the Calculus III class. For that reason we do not prove the result here but will make use of the result. As an example, the Laplace Transform of the convolution of two functions is given below.

Example 3. *Determine the Laplace Transform of $f * g$ where $f(t) = e^t$ and $g(t) = cos(t)$.*

The Laplace Transform of the convolution can be found using Theorem 2,

$$\begin{aligned}\mathcal{L}\{f * g\} &= \mathcal{L}\{f\} \cdot \mathcal{L}\{g\},\\ &= \mathcal{L}\left\{e^t\right\} \cdot \mathcal{L}\{cos(t)\},\\ &= \frac{1}{s-1} \cdot \frac{s}{s^2+1},\\ &= \frac{1}{(s-1)(s^2+1)}.\end{aligned}$$

In the special case of an exponential function the Laplace Transform can be used to determine the convolution without having to perform the integral. The idea is that if you wish to determine the value of the integral

$$I = \int_0^t e^{mw} g(w)\,dw,$$

Then you can multiply by e^{-mt} to get

$$\begin{aligned} Ie^{-mt} &= e^{-mt} \int_0^t e^{mw} g(w)\, dw, \\ &= \int_0^t e^{-mt} e^{mw} g(w)\, dw, \\ &= \int_0^t e^{mw-mt} g(w)\, dw, \\ &= \int_0^t e^{m(w-t)} g(w)\, dw, \\ &= \int_0^t e^{-m(t-w)} g(w)\, dw. \end{aligned}$$

The right hand side is the convolution of e^{-mt} and $g(t)$. The value of the integral can be found by taking the Laplace Transform of both sides,

$$\begin{aligned} \mathcal{L}\left\{Ie^{-mt}\right\} &= \mathcal{L}\left\{(e^{-mt}) * g(t)\right\}, \\ &= \mathcal{L}\left\{e^{-mt}\right\} \cdot \mathcal{L}\left\{g(t)\right\}. \end{aligned}$$

The Laplace Transform of the right hand side can now be found. The function on the right can then be found by expanding into a form in which the inverse Laplace Transform can be found. This usually is done through the use of partial fractions. Once the function on the right hand side is determined the value of the function I can be found by multiplying both sides of the equation by e^{mt},

$$\begin{aligned} Ie^{-mt} &= L^{-1}\left\{\mathcal{L}\left\{e^{-mt}\right\} \cdot \mathcal{L}\left\{g(t)\right\}\right\}, \\ I &= e^{mt} L^{-1}\left\{\mathcal{L}\left\{e^{-mt}\right\} \cdot \mathcal{L}\left\{g(t)\right\}\right\}. \end{aligned}$$

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Kelly Black is a numerical analyst with an interest in education. He likes to keep students busy with writing, online activities, and mathematical modeling. He is also curious about ways to better utilize webworks. His favorite classes to teach are the introductory courses in the students' first 2 years. In his spare time he enjoys cycling and fly fishing.

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